BTP : Shepherding of multievader system with optimal pursuer trajectory

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Overview

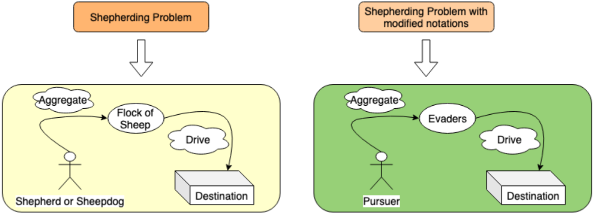
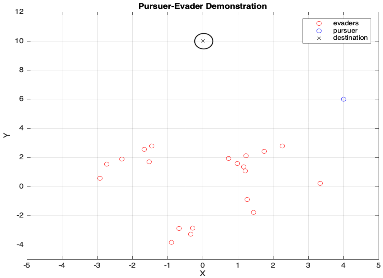


Figure: Overview of shepherding problem



General Problem Description

Assumptions

- N_e evaders and N_p pursuers located in a 2-dim-space (can be extended to N-dim-space)
- Each agent (pursuer as well as evader) has knowledge about the state of all other agents

Task of the Pursuer

- Drive the evaders to an ϵ radius of a predefined fixed destination point z based on some interaction rules between the pursuer and evaders
- Try to minimize the trajectory length of the pursuer

Interaction Rules

1. Repulsion force acting on evaders due to the pursuer $= F_r$
2. Attractive force on an evader towards the centroid of its n-nearest evaders $= F_a$
3. Repulsion acting between two evaders when $\|d(e_i, e_j, t)\|_2 \leq r_a$ (some threshold) $= f_r$
4. Thus, total force acting on an evader $F = F_r + F_a + f_r$

Common Notations

Below are some common notations which will be used throughout this presentation

- N_e denotes the number of evaders (≥ 1)
- N_p denotes the number of pursuers (=1)
- Let $p(t) \in R^2$ denote the position of the pursuer at time instant t
- Let $e_i(t) \in R^2$ denote the position of the i^{th} evader at time instant t
- Let $z(t) = z \in R^2$ denote the predefined fixed destination point
- $d(e_i, p, t) = e_i(t) - p(t)$ is the line of sight vector pointing from the pursuer towards the i^{th} evader at time instant t
- v_{emax} denotes maximum velocity of an evader
- v_{pmax} denotes maximum velocity of the pursuer
- v_{pmin} denotes minimum velocity of the pursuer
- T denotes the time when the objective has been achieved

Mathematical Formulation

Objective Function

$$\underset{p(t)}{\text{minimize}} \int_0^T \sqrt{\dot{p}(t)^T \dot{p}(t)} dt$$

Constraints

$$\|e_i(T) - z\|_2 \leq \epsilon, \quad i = 1, \ldots, N_e \tag{1}$$

$$v_{pmin} \leq \|\dot{p}(t)\|_2 \leq v_{pmax}, \quad 0 < t < T \tag{2}$$

$\dot{e}_i(t)$ = velocity component due to repulsion from pursuer $(ve_{repl}(t))+$ velocity component due to attraction towards centroid $(ve_{attr}(t))$

$$ve_{repl}(t) = v_{emax\_repl} \exp(-k_1 \|d(e_i, p, t)\|_2) \frac{(1 + cos\theta_{i,t})}{2} \frac{d(e_i, p, t)}{\|d(e_i, p, t)\|_2}, \quad i = 1, \ldots, N_e, 0 < t < \tag{3}$$

$cos\theta_{i,t} = \frac{\dot{p}(t).d(e_i,p,t)}{\|\dot{p}(t)\|_2 \|d(e_i,p,t)\|_2}$, which represents the angle between pursuer velocity vector $\dot{p}(t)$ and pursuer-evader line of sight vector $d(e_i, p, t)$

Constraints (continued)

$$ve_{attr}(t) = v_{emax\_attr} \exp(-k_1 \|d(e_i, p, t)\|_2 + k_2 \|d(c_{i,n}, e_i, t)\|_2) \frac{d(c_{i,n}, e_i, t)}{\|d(c_{i,n}, e_i, t)\|_2} \quad (4)$$

$c_{i,n}$: centroid of n-nearest neighbour of i^{th} evader
$k_1, k_2 \geq 0$

- Conceptually, $\|ve_{attr}(t)\|_2 \propto \frac{1}{\|d(e_i,p,t)\|_2}$. Also, $\|ve_{attr}(t)\|_2 \propto \|d(c_{i,n}, e_i, t)\|_2$
- $\|ve_{attr}\|_2$ can be exponentially exploding. To make it stable, take $k_2 << k_1$ or $k_2 = 0$
- When N_e is large and $F = F_r$, driving the evaders to the destination is difficult
- But when N_e is small and $F = F_r$, only the repulsive force on evaders by pursuer may suffice to drive them towards destination

Mathematical Formulation

Objective Function

$$\underset{p(t)}{\text{minimize}} \int_0^T \sqrt{\dot{p}(t)^T \dot{p}(t)} dt \tag{5}$$

Constraints

$$\|e_i(T) - z\|_2 \leq \epsilon, \quad i = 1, \dots, N_e \tag{6}$$

$$v_{pmin} \leq \|\dot{p}(t)\|_2 \leq v_{pmax}, \quad 0 < t < T \tag{7}$$

$$\dot{e}_i(t) = ve_{repl}(t) = v_{emax\_repl} \exp(-k_1 \|d(e_i, p, t)\|_2) \frac{(1 + cos\theta_{i,t})}{2} \frac{d(e_i, p, t)}{\|d(e_i, p, t)\|} \tag{8}$$

$cos\theta_{i,t} = \frac{\dot{p}(t).d(e_i,p,t)}{\|\dot{p}(t)\| \|d(e_i,p,t)\|}$

Algorithm

Keeping the pursuer speed to be fixed : $\|\dot{p}(t)\|_2 = v_p(t) = c$ (constant)
At each time step t, the pursuer moves in that direction which minimizes the below cost function

$$J(t) = \alpha[\|d(e_1, z, t+1)\|_2 + \|d(e_2, z, t+1)\|_2] + (1-\alpha)\|d(e_1, e_2, t+1)\|_2 \tag{9}$$

$$\underset{\hat{v}_p(t)}{\text{minimize}} \quad J(t)$$

Repeat until $(\|d(e_1, z, t)\|_2 \leq \epsilon$ and $\|d(e_2, z, t)\|_2 \leq \epsilon)$

- Estimate the direction $\hat{\theta}$ in which the pursuer should move such that the cost function is minimized
- Move the pursuer in the direction $\hat{\theta}$

Results

- Cost function decreases and then saturates at some finite non-zero value
- This approach requires a lot of tweaking of parameters such as α, constant speed (c), etc. on case-to-case basis

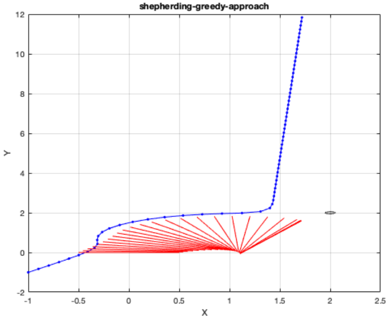


Figure: Shepherding Greedy Approach

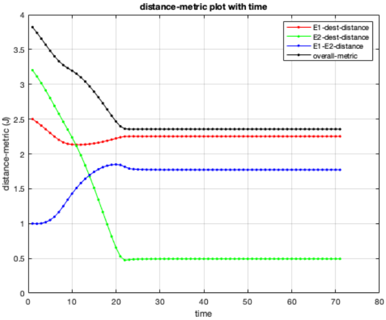


Figure: Plot of cost function and its components with time

Problem Flowgraph

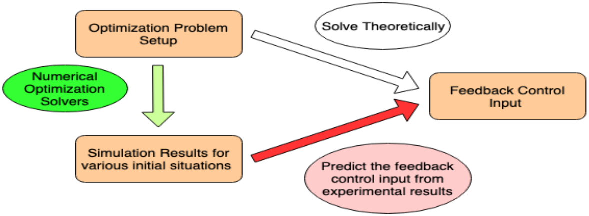


Figure: Problem Flowgraph

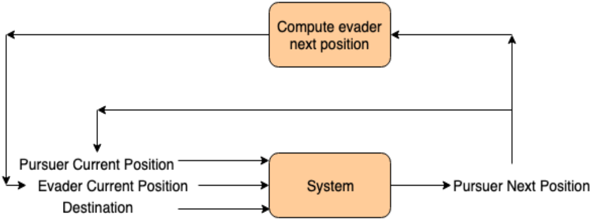


Figure: System Input-Output

Initial Condition

IPP : (-1,-1), Z : (-2,-2)

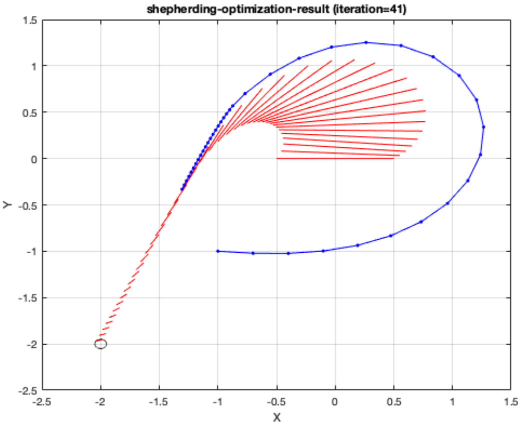


Figure: pursuer-evader trajectories

Initial Condition

IPP : (0,-2), Z : (2,-2)

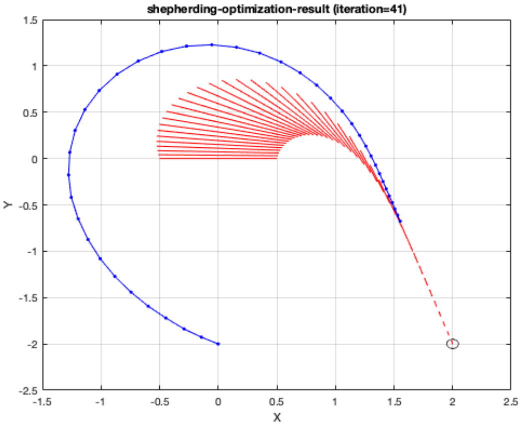


Figure: pursuer-evader trajectories

Initial Condition

IPP : (-2,0), Z : (-2,2)

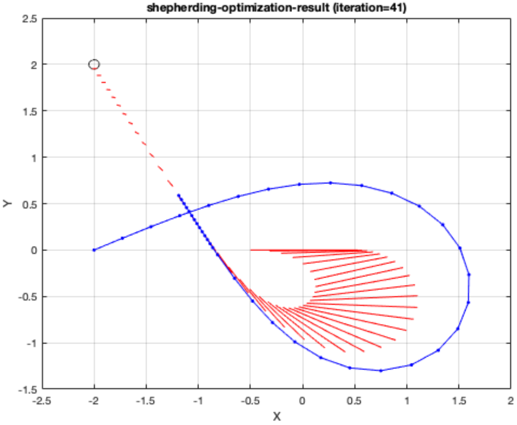


Figure: pursuer-evader trajectories

Initial Condition

IPP : (-1,-1), Z : (1,0.4)

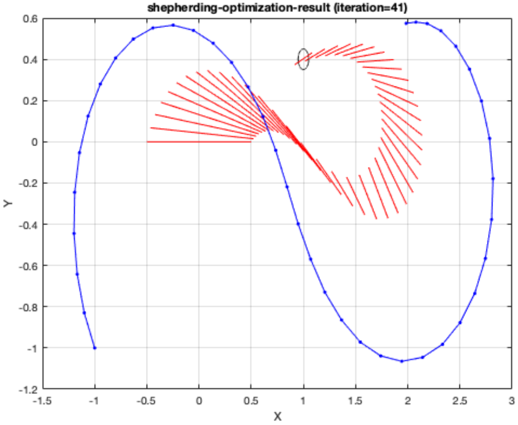


Figure: pursuer-evader trajectories

Initial Condition

IPP : (-2,0), Z : (2,2)

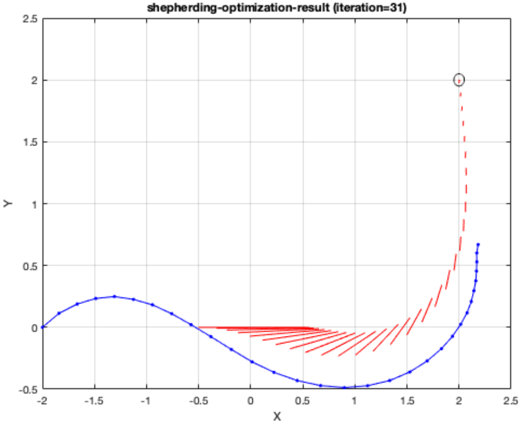


Figure: pursuer-evader trajectories

- Based on the above results, we can think of the two evader-system as a mass-dipole system
- The strength of the dipole can be modeled as magnitude of separation between the two evaders $= \|d(e_1, e_2, t)\|_2$
- The direction of the dipole can be taken as parallel or perpendicular to the evader-separation vector $= \hat{d}(e_1, e_2, t)$

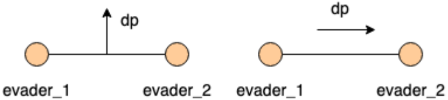


Figure: dipole demontration

- More intuition about the feedback input can be concluded by analyzing the evader trajectory in terms of dipole linear motion and alignment motion (rotation)

- The pursuer's tendency is to bring itself as well as the evaders in a straight line (approx.) with the destination

How to find or predict the final collapsing line based on initial conditions ? Can it be done ? How can this collapsing line be helpful ?

Triangle ΔPZE_c collapses nearly to a single line

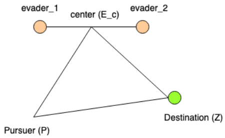


Figure: PZEc Triangle

- **Far Approximation case** : If the pursuer starts very far from the dipole, effect on the dipole in the beginning is negligible
- Significant (noticeable) movement of the evaders happens only when the pursuer is close

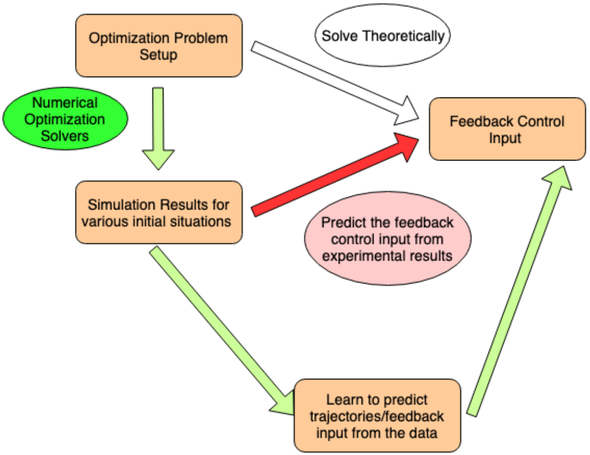


Figure: Problem Flowgraph

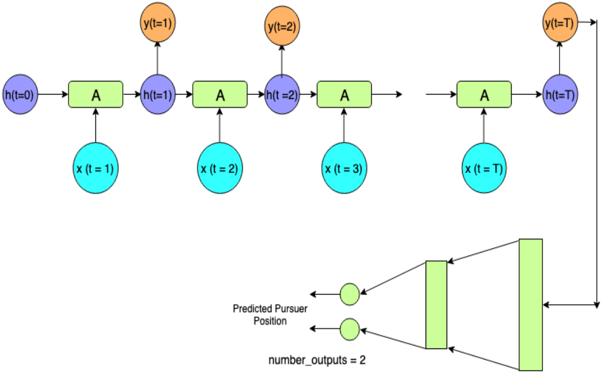


Figure: Model Architecture

RNN Formulation

- RNN state : $h_t = f_w(W_{hh}h_{t-1} + W_{xh}x_t)$, f_w is a non-linear activation function
- RNN output : $y_t = W_{hy}h_t$

Learning model system details & results

- Input features : $x_t = [p_x(t), p_y(t), e^1_x(t), e^1_y(t), e^2_x(t), e^2_y(t), z_x, z_y, \dot{p}_x(t), \dot{p}_y(t), \dot{e}^1_x(t), \dot{e}^1_y(t), \dot{e}^2_x(t), \dot{e}^2_y(t)]$
- mean squared loss $= \frac{1}{M}\sum_{i=1}^{M} \| Y_g(i) - f_\theta(i) \|_2$

Intermediate conclusion on this approach

- The model is able to learn the following details from the data :
  - the smoothness of the trajectories
  - the fact that the pursuer approaches the evaders
  - Approx. the range of the velocity magnitude
- The model fails to learn :
  - the fact that the pursuer has to drive the evaders towards the destination
- Possible Reasons for failure :
  - Insufficient data
  - Model design and complexity (though based on my experience, the model is complex enough to learn these features)

Future Steps

1. Try to formulate a feedback control law for the pursuer based on the observations or develop an iterative solution to achieve the same
2. Generate more simulation results to improve the trajectory accuracy
3. Find out how to improve the learning model in order to put more emphasis on the destination point
4. Things to ponder about :
    1. Dipole Alignment and Linear Motion
    2. Triangle collapse
    3. Estimating the end-collapse line based on initial conditions

Acknowledgement

1. I would like to thank Prof. Debraj for constantly pushing me and giving the necessary directions
2. I would also like to thank Aditya Choudhary for all the help throughout this project

Thank you all for coming :)